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A Note on Zeta Functions Associated with Certain Prehomogeneous Affine Spaces

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Dedicated to Prof. Ichiro Satake on his sixtieth birthday

§ 0. Introduction

The theory of zeta functions associated with prehomogeneous vector spaces (briefly P.V.) was founded by M. Sato and studied by several authors. In [5], Sato and Shintani established the analytic continuation and the functional equations of zeta functions associated with irreducible P.V.'s, and F. Sato [4] extended their results to regular P.V.'s under some mild assumptions. (See also Shintani [6, 7] and Suzuki [8].)

In this paper we shall study some zeta functions associated with *non-regular* P.V. in a special case and prove their analytic continuation and functional equations. Recall that the dual of a regular P.V. is also a regular P.V. and that functional equations hold between zeta functions associated with a regular P.V. and its dual. In our case, however, the dual of our non-regular P.V. is not even a P.V. Thus, instead of the dual, we are led to introduce some prehomogeneous *affine* spaces, the precise definition of which is given below.

Let G be a complex connected linear algebraic group, V a finite dimensional vector space and ρ a rational homomorphism from G into the group of affine transformations of V. We call a triple (G, ρ, V) a *prehomogeneous affine space* (briefly P.A.) if there exists a proper algebraic subset S of V such that $V-S$ is a single G-orbit. The set S is called the *singular set* of (G, ρ, V). In particular, when $\mathrm{Im}\,\rho$ is contained in $GL(V)$, such a triple is called a *prehomogeneous vector space* (briefly P.V.).

In § 1, we define a non-reductive algebraic group $\boldsymbol{G}$ and introduce a pair of non-regular P.V. and P.A. with $\boldsymbol{G}$-action. Zeta functions associated with them are defined and studied in § 2. Though our P.V. and P.A. are not dual to each other, we can prove functional equations between these two types of zeta functions. The next section (§ 3) is devoted to the preparation for the last one. Finally we prove that some contribution to the

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