Advanced Studies in Pure Mathematics 15, 1989 Automorphic Forms and Geometry of Arithmetic Varieties pp. 211-280

Zeta Functions of Finite Graphs and Representations of *p*-Adic Groups

Ki-ichiro Hashimoto*)

Dedicated to Prof. Friedrich Hirzebruch and Prof. Ichiro Satake on their sixtieth birthdays

Contents

- § 0. Introduction
- § 1. Graphs and multigraphs
- § 2. Zeta functions of finite graphs
- § 3. Spectrum of a finite multigraph
- § 4. Harmonic functions and the Hodge decomposition
- § 5. Representations of $C[T_1, T_2]$; a proof of (3.14)
- § 6. Representations of p-adic groups
- § 7. Special values of $Z_x(u)$
- § 8. Miscellaneous results
- § 9. Computations of $Z_x(u)$ for well known families of X
- § 10. Examples: list of graphs for $n \le 6$, $m \le 7$.
- § 11. References

§ 0. Introduction

0-1. In this paper we shall be concerned with the two different subjects, which have been developed separately. One is a combinatorial problem in algebraic graph theory, and the other is an arithmetic of discrete subgroups of p-adic groups and their representations.

Suppose that X is a finite (multi)graph, which is not a tree. We always assume that X is non-oriented. A closed path C in X is called reduced, if C and $C^2 = C.C$ have no backtracking. Then obviously the set $\mathscr{C}_l^{red}(X)$ of reduced closed paths of length l is finite, and $\sharp(\mathscr{C}_l^{red}(X)) \rightarrow \infty$ $(l \rightarrow \infty)$ if X is not homotopic to a circuit, i.e., S^1 . (See § 1 for

Received November 30, 1987.

Revised April 1, 1988.

^{*)} The author has been supported by Sonderforschungsbereich 170, 'Geometrie und Analysis' at Univ. Göttingen.