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Special Values of L-functions Associated with the Space of Quadratic Forms and the Representation of $Sp(2n, F_p)$ in the Space of Siegel Cusp Forms

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Dedicated to Professor I. Satake and Professor F. Hirzebruch for their sixtieth birthdays

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Introduction

0.1. To evaluate special values of various kinds of zeta functions and *L*-functions and to interpret the meaning of them have been providing fruitful problems to number theory.

Siegel [21], as an initiative work, established an ingenious method of

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