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Eisenstein Series on Semisimple Symmetric Spaces of Chevalley Groups

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§ 0. Introduction

0.1. In recent years a remarkable progress has been made in the theory of harmonic analysis on (not necessarily Riemannian) semisimple symmetric spaces by Oshima, Flensted-Jensen and others (cf. [7], [15]). It is also interesting to investigate semisimple symmetric spaces from the arithmetic point of view. For example, in a previous paper [21], we associated with an arbitrary indefinite rational symmetric matrix a family of Dirichlet series satisfying certain functional equations which can be regarded as Eisenstein series on the non-Riemannian symmetric space $SL(n;\boldsymbol{R})/SO(p, n-p)$. This result, along with the recent development in the theory of semisimple symmetric spaces, leads us to the problem of constructing an analogue of Eisenstein series for arbitrary semisimple symmetric spaces with $\boldsymbol{Q}$-structure. Though it seems fairly difficult to solve the problem in its full generality, we are able to find a solution in some special cases. In the present paper, we treat the case of symmetric spaces of ε-involution type (which was introduced by Oshima and Sekiguchi [15]) of Chevalley groups.

0.2. Now we shall sketch the result in this paper. Let G be a connected and simply connected semisimple algebraic group defined and split over $\boldsymbol{Q}$ and σ be an involutive automorphism of G defined over $\boldsymbol{Q}$. Denote by H the fixed point group of σ. A torus T of G is said to be σ-anisotropic if $\sigma(t)=t^{-1}$ for any $t\in T$. We consider the symmetric space $X=G/H$ under the assumption that G has a $\boldsymbol{Q}$-split σ-anisotropic maximal torus. Then every $G_{\boldsymbol{R}}$-orbit in $X_{\boldsymbol{R}}$ is a semisimple symmetric space of ε-involution type in the sense of Oshima and Sekiguchi [15]. In particular, the Riemannian symmetric space $G_{\boldsymbol{R}}/K$ appears among $G_{\boldsymbol{R}}$-orbits in $X_{\boldsymbol{R}}$ for an appropriate choice of σ.

Fix a $\boldsymbol{Q}$-split σ-anisotropic maximal torus T and let B be a Borel subgroup of G such that $B\cap\sigma(B)=T$. Let Φ^+ (resp. Δ) be the set of

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