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Every 3-Manifold Admits a Transverse Pair of Codimension One Foliations Which Cannot be Raised to a Total Foliation

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§ 1. Introduction

Let M be an n-dimensional C^∞ manifold with or without boundary and let $\mathscr{F}$ be a C^r foliation of codimension k of M $(r \geq 1)$. If $\partial M \neq \emptyset$, then for each connected component $(\partial M)_i$ of ∂M, each leaf of $\mathscr{F}$ is assumed to be *transverse* to $(\partial M)_i$, that is, $T_x\mathscr{F} + T_x\partial M = T_xM$, $x \in (\partial M)_i$, or assumed to be *tangent* to $(\partial M)_i$, that is, $T_x\mathscr{F} \subset T_x\partial M$, $x \in (\partial M)_i$, where $T_x\mathscr{F}$ denotes the tangent space of $\mathscr{F}$ at x. In the former case, the restriction of $\mathscr{F}$ to $(\partial M)_i$,

$$\mathscr{F}|_{(\partial M)_i} = \{L \cap (\partial M)_i ; L \in \mathscr{F}\}$$

is a C^r foliation of codimension k (in this case we say $\mathscr{F}$ is *transverse* to $(\partial M)_i$), and in the latter case, $\mathscr{F}|_{(\partial M)_i}$ is a C^r foliation of codimension $k-1$ (in this case we say $\mathscr{F}$ is *tangent* to $(\partial M)_i$). Let $\mathscr{G}$ be another C^r foliation of codimension l. We say $\mathscr{G}$ is *transverse* to $\mathscr{F}$ if at every point $x \in M$, $\dim(T_x\mathscr{F} \cap T_x\mathscr{G}) = \max\{n-k-l, 0\}$. In this case we say $\mathscr{G}$ is a *transverse foliation* for $\mathscr{F}$ or $(\mathscr{F}, \mathscr{G})$ is a *transverse pair* of M. If $(\mathscr{F}, \mathscr{G})$ is a transverse pair, let $\mathscr{F} \Cap \mathscr{G}$ denote $\{F \cap G; F \in \mathscr{F}, G \in \mathscr{G}\}$. Then $\mathscr{F} \Cap \mathscr{G}$ is a C^r foliation of codimension m, where $m = \min\{k+l, n\}$. For each leaf F of $\mathscr{F}$ (resp. G of $\mathscr{G}$), the restriction of $\mathscr{F} \Cap \mathscr{G}$ to F (resp. G) is a C^r foliation of codimension $k' = \min\{n-k, l\}$ (resp. $l' = \min\{k, n-l\}$).

In [13] we classified codimension one foliations transverse to the Reeb component of $S^1 \times D^2$, and using this result we proved that the 3-sphere S^3 has a codimension one foliation which does not admit a transverse foliation of codimension one. Following this result, in [8] and [9] Nishimori investigated foliations transverse to a wider class of foliations of 3-manifolds containing the Reeb component, and he showed many other examples of foliations which admit no transverse foliations. Using a result in [8], Tamura showed that every 3-manifold has a codimension

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