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## Moduli of Stable Sheaves — Generalities and the Curves of Jumping Lines of Vector Bundles on P<sup>2</sup>

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## Introduction

For the last two decades great progress has been made in the problem of moduli of algebraic vector bundles, especially in the framework of the geometric invariant theory by D. Mumford. After fruitful works on curves by D. Mumford, M. S. Narasimhan, P. E. Newstead, S. Ramanan and C. S. Seshadri, etc., F. Takemoto started his attempt to generalize the results to higher dimensional cases. And then we succeeded in constructing moduli spaces of algebraic vector bundles. We know, however, only a few on the structure of each moduli space. The author hopes that it is useful at this moment to publish an expository account on the field.

This note consists of two parts. Part I will treat generalities of the moduli spaces of algebraic vector bundles. Except for a few, almost all are well-known to the experts of this field. In § 1 the meaning of the moduli spaces is explained and we shall show that we can not construct the moduli spaces without restricting ourselves to a suitable subfamily of algebraic vector bundles. § 2 is devoted to showing the results on curves as a good prototype. It should be pointed out that in this case there are a lot of beautiful works other than those which are mentioned in this article. The aim of § 3 is to show the present stage of the problem of moduli of algebraic vector bundles. Unfortunately, for lack of his ability and space, the author could not cover all the results. Especially he excluded those on vector bundles on  $\mathbf{P}^3$  by R. Hartshorne, W. Barth, G. Ellingsrud and S. A. Strømme, etc.

Part II deals, contrary to the general viewpoint of Part I, with the curve of jumping lines of a stable vector bundle of rank 2 on  $\mathbf{P}^2$  with the first Chern class 0, in connection with the moduli spaces of such vector bundles. The curve of jumping lines is defined in § 1. The complete proof of the assertions in this section will be given in [20]. The purpose of § 2 and § 3 is to generalize Barth's work on the  $\theta$ -characteristic associated with a vector bundle of rank 2 on  $\mathbf{P}^2$  to relative cases. We can

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