Chapter 6

V-form Traveling Fronts

In this chapter we study traveling fronts of V-form shapes to bistable reaction-diffusion equations in $\mathbb{R}^2$. V-form traveling fronts have been studied by Ninomiya and myself [36, 37] and Hamel, Monneau and Roquejoffre [21, 22], independently. See also [24, 52, 35]. Here we study them by the method of [36, 37].

6.1 Existence of V-Form Traveling Fronts

We study

$$\begin{aligned} &u_t = \Delta u + f(u), && (x,y) \in \mathbb{R}^2, t > 0, \\ &u(x,y,0) = u_0(x,y), && (x,y) \in \mathbb{R}^2. \end{aligned} \tag{6.1}$$

Here u_0 is a given bounded and uniformly continuous function. In this chapter we assume

(A1) f is of class C^1 in some open interval including $[-1,1]$. It satisfies $f(1) = 0$, $f(-1) = 0$, $f'(-1) < 0$, and $f'(1) < 0$;

(A2) f satisfies $\int_{-1}^{1} f(u)\,\mathrm{d}u > 0$;

(A3) there exist $k > 0$ and Φ such that one has

$$-\Phi''(x) - k\Phi'(x) - f(\Phi(x)) = 0, \qquad x \in \mathbb{R},$$

$$-\Phi'(x) > 0, \qquad x \in \mathbb{R},$$

$$\Phi(-\infty) = 1, \quad \Phi(+\infty) = -1.$$

Let

$$\beta = \frac{1}{2}\min\{-f'(-1), -f'(1)\} > 0.$$

There exists a constant $\delta_* \in (0, 1/4)$ with

$$-f'(s) > \beta \quad \text{if} \quad |s+1| < 2\delta_* \text{ or } |s-1| < 2\delta_*.$$

There exist constants $K_0 > 0$ and $\kappa_0 > 0$ such that one has

$$\max\{|\Phi'(x)|, |\Phi''(x)|, |x\Phi'(x)|\} \le K_0 \exp(-\kappa_0 |x|) \qquad \text{for all } x \in \mathbb{R}.$$

Let $c \in (k, \infty)$ be arbitrarily given. Without loss of generality, we assume that a traveling front is moving to the y-direction. Now we put $z = y - ct$ and $u(x,y,t) = w(x,z,t)$. Denoting $w(x,z,t)$ simply by $w(x,y,t)$, we have

$$w_t - \Delta w - cw_y - f(w) = 0, \quad (x,y) \in \mathbb{R}^2,\ t > 0, \tag{6.2}$$

$$w(x,y,0) = u_0(x,y), \quad (x,y) \in \mathbb{R}^2. \tag{6.3}$$