Chapter 3

Properties of Traveling Front Solutions

For reaction-diffusion equations with bistable nonlinear terms, properties of traveling front solutions have been studied by [14, 10] for instance. The assumption on the nonlinear term f in this chapter is as follows. A nonlinear term f is of class C^1 in some open interval including $[-1, 1]$ and satisfies $f(-1) = 0$, $f(1) = 0$, $f'(-1) < 0$, $f'(1) < 0$. We study

$$\begin{aligned} &\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(u), && x \in \mathbb{R},\ t > 0, \\ &u(x, 0) = u_0(x), && x \in \mathbb{R}. \end{aligned} \tag{3.1}$$

Here u_0 is a given bounded uniformly continuous function from $\mathbb{R}$ to $\mathbb{R}$. Let $u(x, t; u_0)$ be the solution of (3.1). Let

$$\beta = \frac{1}{2} \min \{-f'(-1), -f'(1)\} > 0.$$

Let $\delta_* \in (0, 1/4)$ be small enough so that one has

$$\min_{|s+1| \le 2\delta_*} (-f'(s)) > \beta, \min_{|s-1| \le 2\delta_*} (-f'(s)) > \beta.$$

We put

$$M = \max_{|s| \le 1 + 2\delta_*} |f'(s)|.$$

Now we introduce $y = x - ct$ and $w(y, t) = u(x, t)$. Then we have

$$\mathcal{L}[w] = 0, \qquad y \in \mathbb{R}, t > 0, \tag{3.2}$$

$$w(y, 0) = u_0(y), \qquad y \in \mathbb{R}, \tag{3.3}$$

where

$$\mathcal{L}[w] = w_t - w_{yy} - cw_y - f(w), \qquad y \in \mathbb{R}, t > 0.$$

Let $w(y, t; u_0)$ be the solution of (3.2) and (3.3). We study the following profile equation given by

$$\begin{gathered} -\Phi''(y) - c\Phi'(y) - f(\Phi(y)) = 0, \qquad y \in \mathbb{R}, \\ -\Phi'(y) > 0, \qquad y \in \mathbb{R}, \\ \Phi(-\infty) = 1, \quad \Phi(\infty) = -1. \end{gathered} \tag{3.4}$$

We assume that this profile equation has a solution (c, Φ) in this chapter. We introduce a positive constant σ by

$$\sigma = 1 + \frac{\beta + \max_{|s| \le 1 + 2\delta_*} |f'(s)|}{\beta \min \{-\Phi'(y) \mid y \in \mathbb{R},\ -1 + \delta_* \le \Phi(y) \le 1 - \delta_*\}}. \tag{3.5}$$

The sign of c is determined as follows.