Chapter 1
Overview

Let $\mathbb{RP}^2$ denote the real projective plane and $\mathrm{PGL}(3, \mathbb{R})$ denote the group of projective transformations $\mathbb{RP}^2 \to \mathbb{RP}^2$. A *convex projective surface* is a quotient Ω/Γ, where $\Omega \subset \mathbb{RP}^2$ is an open convex subset and Γ is a discrete (torsion-free) subgroup of $\mathrm{PGL}(3, \mathbb{R})$ that leaves Ω invariant. Examples of convex projective surfaces include Euclidean tori (via the embedding of the Euclidean plane in $\mathbb{RP}^2$ as an affine patch), and hyperbolic surfaces (via the Klein model of the hyperbolic plane). Some convex projective structures on the once-punctured torus are indicated in Figure 1.1.

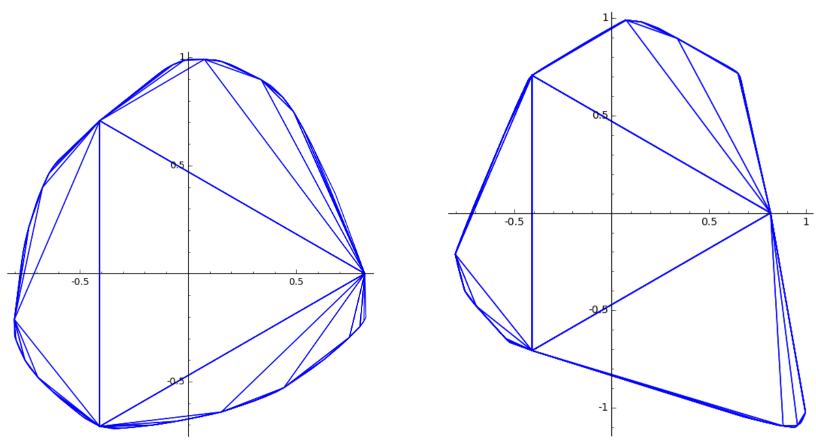

Figure 1.1: Developing maps for convex projective structures on the once-punctured torus [25]

This paper is concerned with *moduli spaces* of convex projective structures on surfaces satisfying additional hypotheses, which will now be described. Let S be a real analytic surface.[1] A *marked properly convex projective structure* on S is a triple (Ω, Γ, f), where:

- $\Omega \subset \mathbb{RP}^2$ is an open subset of the real projective plane which is *proper*, i.e. its closure is contained in an affine patch, and *convex* in that patch;
- Γ is a discrete (torsion-free) subgroup of the group $\mathrm{PGL}(3, \mathbb{R})$ of projective transformations that leaves Ω invariant;
- $f\colon S \to \Omega/\Gamma$ is a real analytic diffeomorphism, where Ω/Γ has a natural real analytic structure, since the action of $\mathrm{PGL}(3, \mathbb{R})$ on $\mathbb{RP}^2$ is real analytic.

[1]Every smooth manifold admits a unique real analytic stiffening [52], and the latter allows us to make use of the concept of a developing map in what is to come.