Introduction

There are different kinds of geometry—they depend on which transformations of space are allowed. Euclidean geometry only allows rigid motions, and this enables us to define distance, angle and volume. In topology, the transformations allowed provide a flexible framework in which the fabric of space is like rubber and which enables the study of the general shape of a space. Our perception of the physical world depends fundamentally on how we imagine the geometry and topology of space-time. This is evidenced by the many physical cosmology theories ranging from Newton, Einstein, and de Sitter's theories to the cyclic models. The objects (or spaces) of geometry and topology are called manifolds, and finding interactions between topological and geometric structures on them holds the key to their study.

To re-imagine the geometry or the topology of a manifold leads to new insights. Some manifolds wear a geometry in one and only one way, while others can wear it in different ways. If one is able to parametrise the different ways in which a geometry is worn, one obtains a *moduli space of geometric structures.*

Given an arbitrary manifold, three fundamental questions are:

- Which geometry should I choose for the manifold?
- How can I parametrise the moduli space of geometric structures?
- What information about the manifold can I gain from the moduli space?

Following Klein's classical viewpoint, the geometry of a manifold is studied via the action of its fundamental group by transformations on a model geometry. This action is called the *holonomy representation*, and deformations of the structure are often studied via the space of conjugacy classes of holonomy representations. One can unroll a manifold in the model geometry just as one can imagine cutting a cylinder and unrolling it in the plane. This leads to a *developing map* for the geometric structure. In the context of these notes, a holonomy representation uniquely determines a developing map for the geometric structure and the so-called Thurston-Ehresmann principle for small deformations applies.

For any 2–dimensional manifold—a surface—classical work of Riemann, Jordan and Möbius from the 19^{th} century gives a clear answer to the first fundamental question, linking the geometry to the topology. A surface has a spherical, Euclidean or hyperbolic structure if its Euler characteristic is positive, zero or negative, respectively. For most surfaces, all closed surfaces for instance, this sufficient condition is also necessary. As a result, hyperbolic geometry (the geometry of constant negative curvature) is the most ubiquitous. The second and third fundamental questions are addressed by Teichmüller theory. Different parametrisations are known for the classical Teichmüller space, which encodes the hyperbolic structures on a surface. For instance, its Fenchel-Nielsen coordinates arise from an explicit construction of