Chapter 2
Preliminaries

In this chapter we give several notation and a quick review of basic concepts which will be used throughout this monograph.

2.1 Notation

2.1.1 Sets of numbers and the Euclidean space.

$\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$ denote the sets of natural numbers, integers, real numbers, and complex numbers, respectively. The symbol i stands for the imaginary unit, namely $i = \sqrt{-1}$. $\mathbb{N}_0$ denotes the set of non-negative integers, that is to say, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. $[x]$ denotes the largest integer not exceeding a real number x.

For $d \in \mathbb{N}$, $\mathbb{R}^d$ is the d-dimensional Euclidean space, and $|x| = (\sum_{j=1}^d x_j^2)^{1/2}$ for $x = (x_1, \ldots, x_d) \in \mathbb{R}^d$. An element of $\mathbb{R}^d$ is sometimes written as $x = (x_1, \ldots, x_d)^{\mathrm{T}}$ when it is convenient, where B^{T} denotes the transpose of a matrix (or vector) B. For $x = (x_1, \ldots, x_d)$, $y = (y_1, \ldots, y_d) \in \mathbb{R}^d$, the inner product $x \cdot y$ is given by $x \cdot y = \sum_{j=1}^d x_j y_j$. For $x \in \mathbb{R}^d$, we set

$$\langle x \rangle = \sqrt{1 + |x|^2}.$$

It is easy to see that $(1 + |x|)/\sqrt{2} \leq \langle x \rangle \leq 1 + |x|$ for any $x \in \mathbb{R}^d$; in other words, $1 + |x|$ and $\langle x \rangle$ are equivalent.

For $R > 0$ and $y \in \mathbb{R}^d$, $\boldsymbol{B}_R(y)$ denotes a closed ball centered at y with radius R, that is to say

$$\boldsymbol{B}_R(y) = \{x \in \mathbb{R}^d;\ |x - y| \leq R\}.$$

We put $\boldsymbol{B}_R := \boldsymbol{B}_R(0)$, where 0 is the origin in $\mathbb{R}^d$.

We write $\mathbb{S}^{d-1}$ for the $(d-1)$-dimensional unit sphere, namely

$$\mathbb{S}^{d-1} = \{x \in \mathbb{R}^d; |x| = 1\}.$$

For any given smooth hypersurface D in $\mathbb{R}^d$, we write dS for its surface element. We use a suffix to indicate the variable of an integral explicitly; namely an expression like $\int_D f(x) dS_x$ or $\int_D f(y) dS_y$ will be used.

2.1.2 Differentiation and the multi-indices.

The notion of multi-indices is quite useful to simplify the descriptions. An element $\alpha = (\alpha_1, \ldots, \alpha_d) \in \mathbb{N}_0^d$ is called a *multi-index*. We define

$$|\alpha| := \alpha_1 + \cdots + \alpha_d, \qquad \alpha! := \alpha_1! \cdots \alpha_d!,$$
$$x^\alpha := x_1^{\alpha_1} \cdots x_d^{\alpha_d} \text{ for } x = (x_1, \ldots, x_d) \in \mathbb{R}^d.$$