Chapter 1

Introduction

The minimal model program is sometimes called *Mori's program* or *Mori theory.* This is because Shigefumi Mori's epoch-making paper [Mo2] is one of the starting points of the minimal model program. Therefore, we quickly review Mori's results in [Mo1] and [Mo2] in Section 1.1. In Section 1.2, we explain some basic examples of quasi-log schemes. By using the theory of quasi-log schemes, we can treat log canonical pairs, non-klt loci of log canonical pairs, semi-log canonical pairs, and so on, on an equal footing. By [F35], the theory of quasi-log schemes seems to be indispensable for the study of semi-log canonical pairs. In Section 1.3, we explain some vanishing theorems, which are much sharper than the usual Kawamata–Viehweg vanishing theorem and the algebraic version of the Nadel vanishing theorem, in order to motivate the reader to read this book. In Section 1.4, we quickly see the main idea of [Am1] and our approach, which is different from the traditional X-method. In Section 1.5, we give several historical comments on this book and the recent developments of the minimal model program for the reader's convenience. We explain the reason of the delay of the publication of this book. In Section 1.6, we compare this book with the unpublished manuscript written and circulated in 2008. In Section 1.7, we quickly review the author's related papers and results for the reader's convenience. In the final section: Section 1.8, we fix the notation and some conventions of this book.

1.1 Mori's cone and contraction theorem

In his epoch-making paper [Mo2], Shigefumi Mori obtained the cone and contraction theorem. It is one of the starting points of Mori's program or the minimal model program (MMP, for short).

Theorem 1.1.1 (Cone theorem)**.** *Let X be a smooth projective variety defined over an algebraically closed field. Then we have the following properties.*

(i) *There are at most countably many* (*possibly singular*) *rational curves C_i on X such that*

$$0 < -(C_i \cdot K_X) \leq \dim X + 1,$$

and

$$\overline{NE}(X) = \overline{NE}(X)_{K_X \geq 0} + \sum \mathbb{R}_{\geq 0}[C_i].$$

Note that $\overline{NE}(X)$ is the Kleiman–Mori cone of X, that is, the closed convex cone spanned by the numerical equivalence classes of effective 1-cycles on X.