Chapter 10

Appendix

In this chapter we gather some basic facts on symplectic basis and symplectic coordinates, see for example [36], [19].

10.1 Symplectic vector space

Definition 10.1.1 *Let S be a finite dimensional vector space over $\mathbb{R}$ $(\mathbb{C})$ and let σ be a non degenerate anti-symmetric bilinear form on S. Then we call S a (finite dimensional) real (complex) symplectic vector space. Let S_i $(i = 1, 2)$ be two symplectic vector spaces with symplectic forms σ_i. If a linear bijection*

$$T : S_1 \to S_2$$

verifies $T^*\sigma_2 = \sigma_1$ then T is called a symplectic isomorphism.

REMARK: σ is said to be non degenerate if

$$\sigma(\gamma, \gamma') = 0, \ \forall \gamma' \in S \Longrightarrow \gamma = 0.$$

$T^*\mathbb{R}^n = \{(x, \xi) \mid x, \xi \in \mathbb{R}^n\}$ is a symplectic vector space with

$$\sigma((x, \xi), (y, \eta)) = \langle \xi, y\rangle - \langle x, \eta\rangle.$$

Proposition 10.1.1 *Let S be a finite dimensional real symplectic vector space. Then the dimension of S is even and there is a symplectic isomorphism*

$$T : S \to T^*\mathbb{R}^n$$

with some n.

Proof: Let e_j, f_j be the unit vector along x_j, ξ_j axis in $T^*\mathbb{R}^n$ respectively. It is clear that

$$\sigma(e_j, e_k) = \sigma(f_j, f_k) = 0, \ \ \sigma(f_j, e_k) = \delta_{jk} \tag{10.1.1}$$