Chapter 7

Behavior of bicharacteristics

7.1 Results

In this chapter we prove the next result which was proved in [41] for the case that the codimension of Σ is 3 and in [3], [47] in full generality.

Theorem 7.1.1 ([41], [3], [47]) *The following assertions are equivalent.*

(i) *p admits an elementary decomposition,*

(ii) *there is no null bicharacteristic of p issuing from a simple characteristic having a limit point in Σ.*

Thanks to Lemma 3.1.1 it suffices to prove

Theorem 7.1.2 ([3], [47]) *There exists a null bicharacteristic having a limit point in the doubly characteristic set if the condition* (i) *in Theorem 3.5.1 fails.*

In [41], the existence proof of such a null bicharacteristic is based on a peculiarity of 2-dimensional autonomous system. Since such a null bicharacteristic, if exists, is essentially unique it seems to be hard to show the existence of such bicharacteristic by topological arguments as in [41] when the codimension of Σ is greater than 3. In [47] we take a completely different method which we follow here assuming

$$\mathrm{Tr}^+ F_p(\rho) = 0, \quad \rho \in \Sigma. \tag{7.1.1}$$

We refer to [47] for the proof of general case without assuming (7.1.1).

The condition (7.1.1) implies

Lemma 7.1.1 *Assume* (7.1.1)*. Then we have*

$$\mathrm{rank}\,(\{\phi_i, \phi_j\}(\rho))_{0\le i,j\le r} = 2. \tag{7.1.2}$$