Chapter 5

Noneffectively hyperbolic Cauchy problem II

5.1 C^∞ well-posedness

We continue to assume that $\Sigma = \{(x,\xi) \mid p(x,\xi) = 0, dp(x,\xi) = 0\}$ is a C^∞ manifold and (4.1.1) is verified. In this chapter we study the case

$$\mathrm{Ker} F_p^2(\rho) \cap \mathrm{Im} F_p^2(\rho) \neq \{0\}. \tag{5.1.1}$$

As we have seen in Theorem 3.5.1 the following two assertions are equivalent

(i) $H_S^3\, p(\rho) = 0$, $\rho \in \Sigma$,

(ii) p admits an elementary decomposition at every $\rho \in \Sigma$

where S is any smooth function verifying (3.4.1) and (3.4.2). As we shall prove in Chapter 7, the condition (ii) is still equivalent to

(5.1.2) there is no null bicharacteristic of p having a limit point in Σ.

In this chapter we discuss the C^∞ well-posedness of the Cauchy problem assuming (5.1.2) (equivalently assuming (i) in Theorem 3.5.1) under the strict Ivrii-Petkov-Hörmander condition.

Theorem 5.1.1 *Assume* (4.1.1), (5.1.1), (5.1.2) *and the subprincipal symbol P_{sub} verifies the strict Ivrii-Petkov-Hörmander condition on Σ. Then the Cauchy problem for P is C^∞ well posed.*

Let fix any $\rho \in \Sigma$. Thanks to Proposition 3.5.1 near ρ we have an elementary decomposition of $p = -\xi_0^2 + \sum_{j=1}^r \phi_j^2$ such that

$$p = -(\xi_0 + \lambda)(\xi_0 - \lambda) + Q$$