Chapter 3

Noneffectively hyperbolic characteristics

3.1 Elementary decomposition

In what follows we assume that the doubly characteristic set

$$\Sigma = \{(x,\xi) \mid p(x,\xi) = dp(x,\xi) = 0\}$$

of p is a smooth conic manifold. In this section we study p of the form

$$p = -\xi_0^2 + a_1(x,\xi')\xi_0 + a_2(x,\xi')$$

which is hyperbolic with respect to ξ_0.

Definition 3.1.1 *We say that $p(x,\xi)$ admits an elementary decomposition if there exist real valued symbols $\lambda(x,\xi')$, $\mu(x,\xi')$, $Q(x,\xi')$ defined near $x = 0$, depending smoothly on x_0, homogeneous of degree 1, 1, 2 respectively and $Q(x,\xi') \geq 0$ such that*

$$p(x,\xi) = -\Lambda(x,\xi)M(x,\xi) + Q(x,\xi'),$$
$$\Lambda(x,\xi) = \xi_0 - \lambda(x,\xi'),\ M(x,\xi) = \xi_0 - \mu(x,\xi'),$$
$$|\{\Lambda(x,\xi), Q(x,\xi')\}| \leq CQ(x,\xi'), \tag{3.1.1}$$
$$|\{\Lambda(x,\xi), M(x,\xi)\}| \leq C(\sqrt{Q(x,\xi')} + |\Lambda(x,\xi') - M(x,\xi')|) \tag{3.1.2}$$

with some constant C. If we can find such symbols defined in a conic neighborhood of ρ then we say that $p(x,\xi)$ admits an elementary decomposition at ρ.

Lemma 3.1.1 ([26]) *Assume that p admits an elementary decomposition. Then there is no null bicharacteristic which has a limit point in Σ.*