Chapter 2

Hyperbolic double characteristics

2.1 Hamilton map

Let us denote by $T^*\Omega$ the cotangent bundle over Ω with a system of local coordinates $x = (x_0, x_1, ..., x_n)$. Let (x, ξ) be a system of canonical coordinates on $T^*\Omega$, then the canonical 2-form σ on $T^*\Omega$ is given by

$$\sigma = \sum_{j=0}^{n} d\xi_j \wedge dx_j$$

in these coordinates. This 2-form gives a symplectic structure on $T^*\Omega$.

Let $f \in C^\infty(T^*\Omega)$. Then the Hamilton vector field H_f of f is defined by

$$df(\cdot) = \sigma(\cdot, H_f). \tag{2.1.1}$$

In the canonical coordinates (x, ξ), denoting $X = \alpha\partial/\partial x + \beta\partial/\partial\xi$, $H_f = a\partial/\partial x + b\partial/\partial\xi$ we have

$$df(X) = \alpha\frac{\partial f}{\partial x} + \beta\frac{\partial f}{\partial \xi} = d\xi \wedge dx(\alpha\frac{\partial}{\partial x} + \beta\frac{\partial}{\partial \xi}, H_f) = \langle\beta, a\rangle - \langle\alpha, b\rangle.$$

That is $b = -\partial f/\partial x$, $a = \partial f/\partial\xi$ and hence

$$H_f = \frac{\partial f}{\partial \xi}\frac{\partial}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial}{\partial \xi}.$$

It is clear that

$$\sigma((x, \xi), (y, \eta)) = \langle\xi, y\rangle - \langle x, \eta\rangle$$

in a system of canonical coordinates.

Let $P(x, D)$ be a differential operator of order m on Ω and let

$$P(x, D) = P_m(x, D) + P_{m-1}(x, D) + \cdots .$$