# Chapter 5

# Topology of 2-orbifolds: 2-orbifold topological constructions

We now wish to concentrate on 2-orbifolds to illustrate more concretely. In many cases, the theory is much easier to understand. Also, we study the topological constructions of 2-orbifolds. We will follow the papers [Choi and Goldman (2005); Scott (1983)].

We first classify smooth 2-orbifolds with possibly empty boundary up to diffeomorphisms. Next 1-dimensional suborbifolds are classified. We discuss the Euler characteristic and the Riemann-Hurwitz formula. We classify the bad orbifolds by discussing about the good, very good, and bad 2-orbifolds. (At present, we can do this for 2-orbifolds only. For higher dimensions, these may not be appropriate terminologies even.)

In the rest of the chapter, we discuss topological cut-and-paste methods applicable to 2-orbifolds.

## 5.1 The properties of 2-orbifolds

Recall that the singular points of a two-dimensional orbifold fall into three types (See Figure 4.7):

(i) The mirror point: $\mathbb{R}^2/\mathbb{Z}_2$ where $\mathbb{Z}_2$ acts by reflections on the $y$-axis.
(ii) The cone-points of order $n$: $\mathbb{R}^2/\mathbb{Z}_n$ where $\mathbb{Z}_n$ acting by rotations by angles $2\pi m/n$ for integers $m$.
(iii) The corner-reflector of order $n$: $\mathbb{R}^2/D_n$ where $D_n$ is the dihedral group generated by reflections about two lines meeting at an angle $\pi/n$.

From this, we obtain that the underlying space of a 2-orbifold is a surface with corner.

The singular strata associated with conjugate local groups are as follows: a silvered point belongs to a 1-dimensional strata, called a *silvered arc*. The other types have isolated points as strata. Recall that boundary of a 2-orbifold is a suborbifold. The silvered arc may have an end point in the boundary of the 2-orbifold and it may end in a corner-reflector of order $\geq 2$ also but not at a cone-point