Chapter 6

Appendix

6.1 Time local solvability of energy-transport model

We begin the detail discussion on the solvability to the problem (4.3)–(4.6) with studying the linear system of equations for an unknown function $(\hat{v}, \hat{w})$

$$\begin{pmatrix} \hat{v} \\ 3\hat{w}/2 \end{pmatrix}_t - A[v,w] \begin{pmatrix} \hat{v} \\ \hat{w} \end{pmatrix}_{xx} + B[v,w] \begin{pmatrix} \hat{v} \\ \hat{w} \end{pmatrix}_x = F[v,w], \tag{6.1}$$

$$B[v,w] := \begin{pmatrix} -e^w(v_x + w_x) & -e^w(v_x + w_x) \\ -e^w(v_x + w_x) - 5e^w w_x/2 & -e^w(v_x + w_x) - 5e^w w_x/2 - \kappa_0 e^{-v} w_x \end{pmatrix},$$

$$F[v,w] := \begin{pmatrix} -e^v + D - v_x(\Phi[e^v])_x \\ -e^v + D - (\Phi[e^v])_x\{2v_x + 7w_x/2 - e^{-w}(\Phi[e^v])_x\} - 3(1 - e^{-w})/2\zeta \end{pmatrix},$$

where A and Φ are given in (2.8) and (4.3b), respectively. The equation (6.1) is a linearization of (4.3). We prescribe the initial condition (4.4) and the boundary conditions (4.5) and (4.6).

The coefficients (v, w) in (6.1) are functions satisfying

$$v, w \in \mathfrak{Z}([0,T]) \cap \mathfrak{Y}_{loc}((0,T)) \tag{6.2}$$

and the estimates

$$\|(v - \Xi, w)(t)\|_1^2 \le M_1, \quad \Xi(x) := (1-x)\log\rho_l + x\log\rho_r, \tag{6.3a}$$

$$\int_0^t \|(v_t, w_t, v_{xx}, w_{xx})(\tau)\|^2 \, d\tau \le M_2, \tag{6.3b}$$

$$t\|(v_t, w_t, v_{xx}, w_{xx})(t)\|^2 + \int_0^t \tau\|(v_{xt}, w_{xt})(\tau)\|^2 \, d\tau \le M_3 \tag{6.3c}$$