Chapter 5

Hydrodynamic model

This chapter is devoted to the proofs of Theorems 2.1 and 2.3. Theorem 2.1 shows the unique existence and the asymptotic behavior of the time global solution to the hydrodynamic model for the large initial data. It is summarized in Theorem 2.3 that the time global solution for this model convergences to that for the energy-transport model as the parameter ε tends to zero. Theorem 2.1 is proven in Sections 5.1–5.3. The uniform estimate of the time local solution, of which existence is discussed in Appendix 6.2, is established in Section 5.1. The time global existence is proven in Sections 5.2 and 5.3. These discussions are essentially same as those for the energy-transport model in Sections 4.1–4.3. The relaxation limit of the solution thus constructed for the hydrodynamic model is studied in Section 5.4, which completes the proof of Theorem 2.3.

5.1 Uniform estimate of local solution

We show that there exists a positive constant T_*, independent of ε, such that the initial boundary value problem (2.11), (2.12) and (2.4)–(2.6) has a solution until time T_*. It is summarized in Corollary 5.3. To show it, we firstly prove the following lemma, which asserts the existence of the time local solution even though the existence time T_ε may depend on ε. Its proof is postponed until Appendix 6.2.

Lemma 5.1. *Suppose the initial data* $(\rho_0, j_0, \theta_0) \in H^2(\Omega) \times H^2(\Omega) \times H^3(\Omega)$ *and the boundary data* ρ_l, ρ_r *and* ϕ_r *satisfy* (2.4), (2.6), (2.7), (2.10a), (2.10b) *and* (2.13). *Let* n *and* N *be certain positive constants satisfying*

$$\inf \rho_0,\ \inf \theta_0,\ \inf S[\rho_0, j_0, \theta_0] \geq n, \quad \|(\rho_0, j_0)(t)\|_2 + \|\theta_0\|_3 \leq N,$$

respectively. Then there exists a positive constant T_ε, *depending on* ε, n *and* N, *such that the initial boundary value problem* (2.11), (2.12) *and* (2.4)–(2.6) *has a unique solution* (ρ, j, θ, ϕ)