Chapter 4

Energy-transport model

This chapter is devoted to showing Theorem 2.4, which asserts that the time global solution for the energy-transport model converges to that for the drift-diffusion model as the parameter ζ tends to zero. The proof is discussed in several sections. We firstly prove in Sections 4.1–4.3 the existence of the time global solution for the energy-transport model with the large initial data $(\rho_0, \theta_0) \in H^1(\Omega)$, which is summarized in Theorem 4.2. The relaxation limit from the energy-transport model to the drift-diffusion model is justified in Section 4.4. These discussion complete the proof of Theorem 2.4.

The unique existence of the time global solution $(\rho_0^0, j_0^0, \phi_0^0)$ for the drift-diffusion model with the initial data $\rho_0 \in H^2(\Omega)$ has been shown in Theorem 2.4 in the authors' previous paper [33]. This result is, however, insufficient in the present paper as we take the initial data $(\rho_0, \theta_0) \in H^1(\Omega)$ to consider the relaxation limit. Hence we show the time global solvability of the model for $\rho_0 \in H^1(\Omega)$ in the next lemma by applying Theorem 2.4 in [33]. Here and hereafter, we use the function spaces

$$\begin{aligned}
\mathfrak{Y}([0,T]) &= C^1([0,T]; L^2(\Omega)) \cap C([0,T]; H^2(\Omega)) \cap H^1(0,T; H^1(\Omega)),\\
\mathfrak{Y}_{loc}((0,T)) &:= C^1((0,T); L^2(\Omega)) \cap C((0,T); H^2(\Omega)) \cap H^1_{loc}(0,T; H^1(\Omega)),\\
\mathfrak{Z}([0,T]) &:= C([0,T]; H^1(\Omega)) \cap L^2(0,T; H^2(\Omega)) \cap H^1(0,T; L^2(\Omega)),\\
\mathfrak{Z}_{loc}((0,T)) &:= C((0,T); H^1(\Omega)) \cap L^2_{loc}(0,T; H^2(\Omega)) \cap H^1_{loc}(0,T; L^2(\Omega)),
\end{aligned}$$

where $\mathfrak{Y}$ is defined in Section 2.3.

Lemma 4.1. *Let $(\tilde{\rho}_0^0, \tilde{j}_0^0, \tilde{\phi}_0^0)$ be the stationary solution to* (2.18)*,* (2.20) *and* (3.2)*. Suppose that the initial data $\rho_0 \in H^1(\Omega)$ and the boundary data ρ_l, ρ_r and ϕ_r satisfy* (2.4)*,* (2.6)*,* (2.7a) *and* (2.10a)*. Then there exists a positive constant δ_0 such that if $\delta \le \delta_0$, the initial boundary value problem* (2.15)*,* (2.12a)*,* (2.4) *and* (2.6) *has a unique solution $(\rho_0^0, j_0^0, \phi_0^0)$ satisfying $\rho_0^0 - \tilde{\rho}_0^0 \in \mathfrak{Z}([0,\infty)) \cap \mathfrak{Y}_{loc}((0,\infty))$, $j_0^0 - \tilde{j}_0^0 \in C([0,\infty); L^2(\Omega))$, $\phi_0^0 - \tilde{\phi}_0^0 \in C([0,\infty); H^3(\Omega)) \cap$*