Chapter 3

Stationary solutions

3.1 Unique existence of stationary solutions

This section is devoted to discussion about the unique existence of stationery solutions for the hydrodynamic, the energy-transport and the drift-diffusion models. We write the solution for the hydrodynamic model by $(\tilde{\rho}^\varepsilon_\zeta, \tilde{j}^\varepsilon_\zeta, \tilde{\theta}^\varepsilon_\zeta, \tilde{\phi}^\varepsilon_\zeta)$ for the clarity of its dependence on ε and ζ. Namely,

$$
\begin{gathered}
(\tilde{j}^\varepsilon_\zeta)_x = 0, \\
S[\tilde{\rho}^\varepsilon_\zeta, \tilde{j}^\varepsilon_\zeta, \tilde{\theta}^\varepsilon_\zeta](\tilde{\rho}^\varepsilon_\zeta)_x + \tilde{\rho}^\varepsilon_\zeta(\tilde{\theta}^\varepsilon_\zeta)_x = \tilde{\rho}^\varepsilon_\zeta(\tilde{\phi}^\varepsilon_\zeta)_x - \tilde{j}^\varepsilon_\zeta, \\
\tilde{j}^\varepsilon_\zeta(\tilde{\theta}^\varepsilon_\zeta)_x - \frac{2}{3}\tilde{j}^\varepsilon_\zeta\tilde{\theta}^\varepsilon_\zeta\left(\log\tilde{\rho}^\varepsilon_\zeta\right)_x - \frac{2}{3}\kappa_0(\tilde{\theta}^\varepsilon_\zeta)_{xx} = \left(\frac{2}{3} - \frac{\varepsilon}{3\zeta}\right)\frac{(\tilde{j}^\varepsilon_\zeta)^2}{\tilde{\rho}^\varepsilon_\zeta} - \frac{\tilde{\rho}^\varepsilon_\zeta}{\zeta}(\tilde{\theta}^\varepsilon_\zeta - 1), \\
(\tilde{\phi}^\varepsilon_\zeta)_{xx} = \tilde{\rho}^\varepsilon_\zeta - D.
\end{gathered}
$$

The stationary solution for the energy-transport model is written by $(\tilde{\rho}^0_\zeta, \tilde{j}^0_\zeta, \tilde{\theta}^0_\zeta, \tilde{\phi}^0_\zeta)$ and satisfies

$$(\tilde{j}^0_\zeta)_x = 0, \tag{3.1a}$$

$$\tilde{j}^0_\zeta(\tilde{\theta}^0_\zeta)_x - \frac{2}{3}\tilde{j}^0_\zeta\tilde{\theta}^0_\zeta\left(\log\tilde{\rho}^0_\zeta\right)_x - \frac{2}{3}\kappa_0(\tilde{\theta}^0_\zeta)_{xx} = \frac{2}{3}\frac{(\tilde{j}^0)^2}{\tilde{\rho}^0} - \frac{\tilde{\rho}^0_\zeta}{\zeta}(\tilde{\theta}^0_\zeta - 1), \tag{3.1b}$$

$$(\tilde{\phi}^0_\zeta)_{xx} = \tilde{\rho}^0_\zeta - D, \tag{3.1c}$$

$$\tilde{j}^0_\zeta = -(\tilde{\theta}^0_\zeta\tilde{\rho}^0_\zeta)_x + \tilde{\rho}^0_\zeta(\tilde{\phi}^0_\zeta)_x \tag{3.1d}$$