CHAPTER 6

The Case of Complex Codimension One

The Godbillon–Vey class of *real* codimension-one foliations is deeply studied. One of the most significant results is due to Duminy.

THEOREM 6.1 (Duminy [27]). *Let $\mathcal{F}$ be a real codimension-one foliation of a closed manifold M. Then,* $\mathrm{GV}_1(\mathcal{F})$ *is non-trivial only if $\mathcal{F}$ admits a resilient leaf.*

A leaf of $\mathcal{F}$ is called *resilient* if the leaf admits a holonomy by which the leaf accumulates on itself. It is known that $\mathcal{F}$ admits a minimal set other than a closed leaf if there is a resilient leaf, and $\mathcal{F}$ has a leaf of exponential growth. This is shown by carefully studying minimal sets.

If the codimension of $\mathcal{F}$ is greater than one, then there is a following analogue shown by Hurder.

THEOREM 6.2 (Hurder [47], [48]). *Let $\mathcal{F}$ be a real codimension-q foliation of a closed manifold M. Suppose that* $\mathrm{GV}_q(\mathcal{F})$ *is non-trivial, then $\mathcal{F}$ admits a leaf of exponential growth.*

This theorem is shown by regarding h_1 as a measure and studying its support.

Compared with the real codimension-one case, study of minimal sets is much more difficult even if $\mathcal{F}$ is of complex codimension-one. There are however useful notions related to complex one-dimensional dynamical systems. For example, there are Julia sets of complex dynamical systems and limit sets of Kleinian groups. These two notions are considered to be of the same nature (Sullivan's dictionary). Roughly speaking, dynamics are complicated on Julia sets or limit sets, and simple