CHAPTER 4

The Rigidity Theorem and Infinitesimal Derivatives

In this chapter, we will introduce infinitesimal derivatives of secondary classes after Heitsch [42]. Deformations of foliations (and pseudogroup structures) are discussed by Kodaira [53], Kodaira–Spencer, [54], Heitsch [40], Duchamp–Kalka [26], Girbau–Haefliger–Sundararaman [31], Girbau–Nicolau [32], et al. It will be shown that complex secondary classes determined by the image of $H^*(\mathrm{WU}_{q+1})$ under the natural mapping to $H^*(\mathrm{WU}_q)$ are rigid under actual and infinitesimal deformations. In particular, the Godbillon–Vey class is shown to be rigid in the category of transversely holomorphic foliations. On the other hand, classes in $H^*(\mathrm{WU}_q)$ which admit continuous deformations are called variable classes. The imaginary part of the Bott class is one of the variable classes. Heitsch introduced in [42] the infinitesimal derivatives for *cocycles* in WU_q which represent variable classes of lowest degree. In the same paper, the infinitesimal derivatives for any *classes* in $H^*(\mathrm{WO}_q)$ were also introduced. The most of this section will be devoted to completing Heitsch's construction by defining the infinitesimal derivatives for any *classes* in $H^*(\mathrm{WU}_q)$. The construction seems known for specialists, indeed, the most of the definitions and the proofs are only small modifications of Heitsch's in [42] using notions in [26]. However, we give the details for completeness and for their importance.

Throughout the construction, corresponding steps or statements in [42] are referred so far as possible.