Chapter 8

Examples and extensions

Theorem 2.4.1 gives estimates for operators provided the characteristic roots satisfy certain hypotheses. However, in order to test the validity of such an estimate for an arbitrary linear, constant coefficient m^{th} order strictly hyperbolic operator with lower order terms, it is desirable to find conditions on the structure of the lower order terms under which certain conditions for the characteristic roots hold. For the case $m = 2$, a complete characterisation can be given, and some extension of this is discussed in Section 8.1. However, for large m, it is difficult to do such an analysis, as no explicit formulae are available in general; nevertheless, certain conditions can be found that do make the task of checking the conditions of the characteristic roots, and these are discussed in Section 8.2, where a method is also given that can be used to find many examples. Finally, in Section 8.5, we give a few applications of these results.

8.1 Wave equation with mass and dissipation

As an example of how to use Theorem 2.4.1, here we will show that we can still have time decay of solutions if we allow the negative mass but exclude certain low frequencies for Cauchy data. This is given in (8.1.1) below. In the case of the negative mass and positive dissipation, there is an interplay between them with frequencies that we are going to exhibit. The usual non-negative and also time dependent mass and dissipation with oscillations have been considered before, even with oscillations. See, for example, [HR03] and references therein.

Let us consider second order equations of the following form

$$\begin{cases} \partial_t^2 u - c^2 \Delta u + \delta \partial_t u + \mu u = 0\,, \\ u(0,x) = 0,\ u_t(0,x) = g(x)\,. \end{cases}$$