Chapter 7

Frequencies around multiplicities

Finally, let us turn to finding estimates for the first term of (6.2.3), which we may write in the form

$$\int_{\Omega} e^{ix\cdot\xi}\Big(\sum_{k=1}^{L} e^{i\tau_k(\xi)t}A_j^k(t,\xi)\Big)\chi(\xi)\widehat{f}(\xi)\,d\xi\,,$$

where the characteristic roots $\tau_1(\xi),\dots,\tau_L(\xi)$ coincide in a set $\mathcal{M}\subset\Omega$ of codimension ℓ (in the sense of Section 2.1), $\Omega\subset\mathbb{R}^n$ is a bounded open set and $\chi\in C_0^\infty(\Omega)$.

As before, we must consider the cases where the image of the phase function(s) either lie on the real axis, are separated from the real axis or meet the real axis. One additional thing to note in this case is that in principle the order of contact at points of multiplicity may be infinite as the roots are not necessarily analytic at such points; we have no examples of such a situation occurring, so it is not worth studying too deeply unless such an example can be found—for now, we can use the same technique as if the point(s) were points where the roots lie entirely on the real axis, and the results in these two situations are given together in Theorem 2.4.1. We study this very briefly nevertheless to ensure the completeness of the obtained results.

Unlike in the case away from multiplicities of characteristic roots, we have no explicit representation for the coefficients $A_j^k(t,\xi)$ (as we have in Lemma 6.1.1 away from the multiplicities), which in turn means we cannot split this into L separate integrals. To overcome this, we first show, in Section 7.1, that a useful representation for the above integral does exist that allows us to use techniques from earlier. Using this alternative representation, it is a simple matter to find estimates in the case where the image of the set $\mathcal{M}$