Chapter 4

Oscillatory integrals with convexity

As discussed in Section 1.2, in the case of homogeneous m^{th} order strictly hyperbolic operators, geometric properties of the characteristic roots play the fundamental role in determining the $L^p - L^q$ decay; in particular, if the characteristic roots satisfy the convexity condition of Definition 1.2.1, then the decay is, in general, more rapid than when they do not. We will show that a similar improvement can be obtained for operators with lower order terms when a suitable 'convexity condition' holds. In Section 4.3, we shall extend this notion of the convexity condition to functions $\tau : \mathbb{R}^n \to \mathbb{R}$ and prove a decay estimate for an oscillatory integral (related to the solution representation for a strictly hyperbolic operator) with phase function τ.

First, we give a general result for oscillatory integrals and show how the concept of functions of "convex type" allows its application to derive the time decay.

4.1 Estimates for oscillatory integrals

The following theorem is central in proving results involving convexity conditions. In some sense, it bridges the gap between the man der Corput Lemma and the method of stationary phase, in that the former is used when there is no convexity but gives a weaker result, while the latter can be used when a stronger condition than simply convexity holds and gives a better result. Here, we state and prove a result that has no reference to convexity; however, in the following section, we show how convexity (in some sense) enables this result to be used in applications. An earlier version of this result has appeared in [Ruzh07], with applications to equations with time dependent