Chapter 3

Properties of hyperbolic polynomials

In order to study the solution $u(t,x)$ to (1.0.1), we must first know some properties of the characteristic roots $\tau_1(\xi), \ldots, \tau_m(\xi)$. Naturally, we do not have explicit formulae for the roots, unlike in the cases of the dissipative wave equation and the Klein–Gordon equation (i.e. for second order equations), but we do know some properties for the roots of the principal symbol. For general hyperbolic operators, the roots $\varphi_1(\xi), \ldots, \varphi_m(\xi)$ of the characteristic polynomial of the *principal part* are homogeneous functions of order 1 since the principal part is homogeneous. Furthermore, for strictly hyperbolic polynomials these roots are distinct when $\xi \neq 0$. Since these two properties are very useful when studying homogeneous (strictly) hyperbolic equations, it is useful to know whether the characteristic roots of the full equation, $\tau_1(\xi), \ldots, \tau_m(\xi)$, have similar properties. Indeed, if we regard the full equation as a perturbation of the principal part by lower order terms, we can show that similar properties hold for large $|\xi|$; these results are the focus of this section. In the outline of the method in Section 2.5, we subdivided the phase space into large $|\xi|$ and bounded $|\xi|$, and it is these properties that motivate this step.

3.1 General properties

First, we give some properties of general polynomials which are useful to us. For constant coefficient polynomials, the following result holds: