Chapter 1

Introduction

This work is devoted to the investigation of dispersive and Strichartz estimates for general hyperbolic equations with constant coefficients. The analysis that we carry out is also applicable to hyperbolic systems either by looking at characteristics of the system directly, or first taking the determinant of the system (the dispersion relation).

There are several important motivations for the analysis. First, while hyperbolic equations of the second order (such as the wave equation, dissipative wave equation, Klein–Gordon equation, etc.) are very well studied, relatively little is known about equations of higher orders. At the same time, equations or systems of high orders naturally arise in applications. For example, Grad systems of non-equilibrium gas dynamics, when linearised near an equilibrium point, are examples of large hyperbolic systems with constant coefficients (see e.g. [Rad03], [Rad05]). Here one has to deal with hyperbolic equations of orders 13, 20, etc., depending on the number of moments in the Grad system. Moreover, there are important families of systems of size going to infinity, or even of infinite hyperbolic systems. For example, the Hermite–Grad method for the analysis of the Fokker–Planck equation for the distribution function for particles for the Brownian motion produces an infinite hyperbolic system with constant coefficients. Indeed, making the decomposition in the space of velocities into the Hermite basis, and writing equations for the space-time coefficients produces a hyperbolic system for infinitely many coefficients (see e.g. [VR03], [VR04], [ZR04], and Section 8.5). The Galerkin approximation of this system leads to a family of systems with sizes increasing to infinity. Although explicit calculations are difficult in these situations, the time decay rate of the solution can still be calculated ([Ruzh06]).

One of the main difficulties when dealing with large systems is that unlike in the case of the second order equations, in general characteristics can not