

Author's Notes (2008)

(I) This is a reproduction of “On Congruence Monodromy Problems” Volume 1 (1968) and Volume 2 (1969) issued as Lecture Notes (No 1,2) from Department of Mathematics, University of Tokyo, which remained unpublished (except for a Russian translation [11] of Volume 1). When this reproduction was proposed, I was sure they were old, for old colleagues, but at the same time not so sure whether they did not contain anything new for the newcomers, because they had not been buried so peacefully. There had been several occasions when I felt I would have liked to share a piece of mathematics in its natural form whose understanding would have clarified my younger colleague's contemporary questions. And on some other occasions, I had to face with some basic misunderstandings¹. So, I agreed that the main results (including those obtained later, which will be briefly reviewed below), some points of view, methods used, and related open problems may still deserve some attention.

I have then re-read the text carefully, and fortunately or unfortunately, found no errors other than small local ones. Aside from these corrections and some arrangements to unify the two Volumes, no changes have been made in the present reproduction. On the other hand, obviously, the author had been too nervous in giving all the details almost everywhere. So, for the possible readers of this reproduction I would suggest reading only the introductions and the outlines, and for details, only when necessary.

This reproduction was proposed by a colleague of mine, Takayuki Oda, who has also taken the labor to put the original typed text into TeX' text (mainly Volume 1). I wish to express my deep gratitude to him and to T.Ichikawa, M.Kaneko and H.Tsutsumi, who had helped a great deal in continuing and finishing this laborious task.

(II) For a given finite Galois extension K/k of global fields, knowing the global Artin L -functions $L(s, \chi, K/k)$ (for all irreducible characters χ of the Galois group) is one thing, knowing the Frobenius conjugacy class for each (unramified) prime of k is quite another. They are the same when k is the rational number field \mathbf{Q} , but *essentially different* for function fields over finite fields. This simple fact does not seem to be widely recognized even in the circle of number theorists, and since the justification of the present reproduction of old lecture notes will never be understood without this recognition, I started with this; now let me recall the reason.

From a global Artin L -function, what one can pick up as local data is, for each prime number p , just *the product of* Euler factors corresponding to those primes of k whose

¹The biggest of which is that (even in the case of function fields) the Langlands correspondence should contain everything related to non-abelian classfield theory and hence our work merely gives its “examples” in disguise. The following (otherwise superfluous) subsection (II) is for some extra guide...