CHAPTER 5

Part 1. Elliptic modular functions mod p and $\Gamma = PSL_2(\mathbf{Z}^{(p)})$.

Our purpose in Part 1 of this chapter is to formulate and prove a fundamental relation between the classes mod $\mathfrak{P}$ ($\mathfrak{P}|p$) of the special values of elliptic modular functions $J(z)$ and the group $\Gamma = PSL_2(\mathbf{Z}^{(p)})$ (Theorems 1, 1′; §5). This is a fruit of

(i) Deuring' s work on complex multiplication of elliptic curves **[4]** **[6]** **[7]**,
(ii) a new standpoint.

Roughly speaking, (ii) is of:
"A fixed p and variable imaginary quadratic fields and lattices",
instead of "a fixed imaginary quadratic field and variable p", which was the standpoint of classical complex multiplication theory. However, besides this new standpoint, nothing more is to be added to Deuring' s work. In fact, the proof of Theorems 1, 1′ based on Deuring's results is quite elementary.

As described in **[18]**, our Theorems 1, 1′ give a starting point of our problems. Generalizations to congruence subgroups of Γ (announced in §10) will be given in Part 2 of this chapter.

Elliptic modular functions mod p and $\Gamma = PSL_2(\mathbf{Z}^{(p)})$.

§1. Throughout this chapter, p is a fixed prime number and Π is the cyclic subgroup of $\mathbf{Q}^\times$ generated by p. Put $\mathbf{Z}^{(p)} = \Pi \cdot \mathbf{Z} = \cup_{n=0}^{\infty} p^{-n}\mathbf{Z}$, and put

(1) $$\Gamma = PSL_2(\mathbf{Z}^{(p)}).$$

It is a discrete subgroup of

$$G = G_\mathbf{R} \times G_p = PSL_2(\mathbf{R}) \times PSL_2(\mathbf{Q}_p).$$

We already know that the quotient G/Γ has finite invariant volume and that $\Gamma_\mathbf{R}, \Gamma_p$ are dense in $G_\mathbf{R}, G_p$ respectively (see Chapter 1, §1, §2). Put

(1*) $$\Gamma^* = \{x \in GL_2(\mathbf{Z}^{(p)}) |\, \det x \in \Pi\} / \pm \Pi.$$