CHAPTER 2

Introduction to Part 1 and Part 2.

Chapter 2 consists of two parts, Part 1 (§1-§17) and Part 2 (§18-§36). The subject here is what we call a "G_p-field", where $G_p = PSL_2(k_p)$. The definition is as follows. A field L is called a G_p-field over a subfield k if $\dim_k L = 1$ and if G_p acts effectively on L as a group of field automorphisms over k, fulfilling the following conditions [1]:

(i) For each open compact subgroup $V \subset G_p$, its fixed field L_V is finitely generated over k, and L/L_V is normally and separably algebraic. Moreover, V is topologically isomorphic to the Krull's Galois group of L/L_V.

(ii) Almost all prime divisors of L_V over k are unramified in L.

(iii) The fixed field of G_p is k. (k is called the *constant field* of L.)

The motivation for the study of such a field is this:
— If Γ is a discrete subgroup of $G = G_{\mathbf{R}} \times G_p$ with finite-volume-quotient such that the projections $\Gamma_{\mathbf{R}}$, Γ_p are dense in $G_{\mathbf{R}}$, G_p respectively, then Γ *defines a G_p-field L over the complex number field $\mathbf{C}$, and conversely* (Theorem 1, §9). Thus Γ *and L (over $\mathbf{C}$) are equivalent notions.* Moreover, it seems that the study of G_p-fields over algebraic number fields [2] is crucial for the solution of our problems. Thus we meet our first problem: *"Is every G_p-field L over $\mathbf{C}$ a constant field extension of a G_p-field L_0 over an algebraic number field ?"* This problem is solved affirmatively in Part 2 (Theorem 4, §18). The readers note, however, that this would not be remarkable enough without *"essential uniqueness" of L_0, which is guaranteed by Theorems 5, 6, 7 (§18, §32, §33) under a certain condition on L.* Namely, by Theorem 5, under a condition on L which is always satisfied if Γ is maximal (see §10), there is a unique [3] G_p-field L_{k_0} over an algebraic number field k_0 such that

(i) L is a constant field extension of L_{k_0}, and

(ii) if L is a constant field extension of another G_p-field L_k over a field $k \subset \mathbf{C}$, then k contains k_0 and $L_k = L_{k_0} \cdot k$.

Thus if Γ is maximal, then Γ defines a unique G_p-field L_{k_0} over an algebraic number field k_0. Theorems 6, 7 are some variations of Theorem 5.

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[1] See also §1. We do not assume that G_p is the full automorphism group of L over k.

[2] By an algebraic number field, we always mean a *finite* extension of the field of rationals $\mathbf{Q}$.

[3] L_{k_0} is unique not only up to isomorphisms, but also as a G_p-invariant subfield of L.