Introduction to Part 3A and Part 3B.[19]

Here, we shall give only rough ideas of problems and results. For the precise formulation of our results, see the main text.

[Indication] To approach e.g., Theorem 10 (§45 [3]), which is one of our main results, the readers are requested to read §37 and §38 for the definition of S-operators, and then §41 [1]~[3] and §45 [1] [2] for the definition of ample fields L/k. (S-operators and ample fields are two main concepts introduced in this study, and are basic for our purpose. G_p-fields are examples of ample fields.)

The Problems . Let $\mathfrak{R}$ be a compact Riemann surface. Suppose that there are given $s+t$ $(0 \le s, t < \infty)$ distinct points $P_1, \dots, P_t; Q_1, \dots, Q_s$ on $\mathfrak{R}$, and s positive integers $e_1, \dots, e_s$ satisfying

$$\text{(i)} \qquad 2g - 2 + t + \sum_{i=1}^{s} \left(1 - \frac{1}{e_i}\right) > 0,$$

where g is the genus of $\mathfrak{R}$. Then, as is well-known, there is a unique simply connected (unbounded) covering $\widetilde{\mathfrak{R}}$ of $\mathfrak{R} \setminus \{P_1, \dots, P_t\}$, isomorphic to the complex upper half plane $\mathfrak{H} = \{\tau \in \mathbf{C} \,|\, \operatorname{Im} \tau > 0\}$, which is unramified except at Q_i and ramified at Q_i with index e_i $(1 \le i \le s)$:

$$\text{(ii)} \qquad \begin{array}{ccl} \widetilde{\mathfrak{R}} & \cong & \mathfrak{H} = \{\tau \in \mathbf{C} \,|\, \operatorname{Im} \tau > 0\} \\ \downarrow & & \\ \mathfrak{R} & - & \{P_1, \cdots, P_t\}. \end{array}$$

Fix an isomorphism $\widetilde{\mathfrak{R}} \simeq \mathfrak{H}$, and consider τ as a multivalued function on $\mathfrak{R} \setminus \{P_1, \dots, P_t\}$. Let $dx \neq 0$ be any meromorphic differential (1-form) on $\mathfrak{R}$ (which may not be exact), and put $\tau_x = \frac{d\tau}{dx}$, $\tau_{\underbrace{xx\dots x}_{i+1}} = \frac{d}{dx}\tau_{\underbrace{xx\dots x}_{i}}$ $(i \ge 1)$, so that $\tau_x, \tau_{xx}, \dots$ are multivalued meromorphic functions on $\mathfrak{R} \setminus \{P_1, \dots, P_t\}$. Put

$$\text{(iii)} \qquad A = -\frac{2\tau_x \cdot \tau_{xxx} - 3\tau_{xx}^2}{\tau_x^2}.$$

Then it is well-known (classically) that A *is a univalent meromorphic function on* $\mathfrak{R}$. Moreover, if we consider A as known, and (iii) as a differential equation for τ, then all the solutions of (iii) are $\frac{a\tau+b}{c\tau+d}$, where $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ are any elements of $GL_2(\mathbf{C})$. Since Aut $\mathfrak{H} = PSL_2(\mathbf{R})$, this shows that A depends only on dx, and is independent of the isomorphism $\widetilde{\mathfrak{R}} \simeq \mathfrak{H}$. So, the map

$$\text{(iv)} \qquad dx \longmapsto A$$

[19]The main contents of these Parts are published in Additional References [16]