Part 1. The $G_\mathfrak{p}$-fields over $\mathbf{C}$.

The $G_\mathfrak{p}$-fields.

§1. Let L be a discrete field, on which the group $G_\mathfrak{p} = PSL_2(k_\mathfrak{p})$ acts effectively and continuously as a group of field-automorphisms; namely, each $g_\mathfrak{p} \in G_\mathfrak{p}$ gives a field automorphism $x \mapsto g_\mathfrak{p}(x)$ of L, and the induced map $G_\mathfrak{p} \to \operatorname{Aut} L$ is an injective homomorphism;

$$(1) \qquad \begin{aligned} &(g_\mathfrak{p} h_\mathfrak{p})(x) = g_\mathfrak{p}(h_\mathfrak{p}(x)) \quad \forall g_\mathfrak{p}, h_\mathfrak{p} \in G_\mathfrak{p},\ x \in L; \\ &g_\mathfrak{p}(x) = x \ \ (\forall x \in L) \leftrightarrow g_\mathfrak{p} = 1. \end{aligned}$$

Since L is a discrete field, the continuity of the actions of $G_\mathfrak{p}$ amounts to saying that, for each $x \in L$, its stabilizer in $G_\mathfrak{p}$ is open. For each open compact subgroup V of $G_\mathfrak{p}$, put

$$(2) \qquad L_V = \{x \in L \mid v(x) = x,\ \forall v \in V\}.$$

Since open compact subgroups form a basis of neighborhoods of the identity of $G_\mathfrak{p}$, we get $L = \bigcup_V L_V$. Moreover, it follows that for each V, L/L_V is separably algebraic, V is the group of *all* automorphisms of L/L_V, and the topology of V induced by that of $G_\mathfrak{p}$ coincides with the Krull topology of $V = \operatorname{Aut}(L/L_V)$. In fact, let $x \in L$, and let V' be its stabilizer in $G_\mathfrak{p}$. Then since V' is open, we have $(V : V' \cap V) < \infty$. Put $V = \sum_{i=1}^{d} \sigma_i(V \cap V')$. Then $\sigma_1(x), \cdots, \sigma_d(x)$ are mutually distinct, and their elementary symmetric functions are all contained in L_V; hence L/L_V is separably algebraic. Now consider $\operatorname{Aut}(L/L_V)$ as equipped with the Krull topology. Then the injection $\varphi : V \to \operatorname{Aut}(L/L_V)$ is continuous, since the action of $G_\mathfrak{p}$ on L is so; hence $\varphi(V)$ is also compact. On the other hand, $\varphi(V)$ is dense in $\operatorname{Aut}(L/L_V)$, since for any $\sigma \in \operatorname{Aut}(L/L_V)$, we have $\sigma(x) = \sigma_i(x)$ for some i (σ_i being as above, for *this* x). Therefore, $\varphi(V) = \operatorname{Aut}(L/L_V)$, and φ is bicontinuous (since V is compact).

Let k be the fixed field of $G_\mathfrak{p}$;

$$(3) \qquad k = \{x \in L \mid g_\mathfrak{p}(x) = x\ \forall g_\mathfrak{p} \in G_\mathfrak{p}\}.$$

We shall call L *a one-dimensional $G_\mathfrak{p}$-field over k,* or simply, a *$G_\mathfrak{p}$-field over* k, if

(L1) $\dim_k L = 1$,

and if for every open compact subgroup V of $G_\mathfrak{p}$, the condition:

(L2) *L_V is finitely generated over k, and almost all prime divisors of L_V over k are unramified in L;*

is satisfied. We note that since L/L_V is algebraic, (L1) implies $\dim_k L_V = 1$; hence L_V is an algebraic function field of one variable over k, in the sense that L_V/k is finitely generated and is of dimension one. By a prime divisor of L_V over k, we mean an equivalence class