2. Cayley graphs.

A Cayley graph gives us a means by which a finitely generated group can be viewed as a geometric object. The starting point is a finite generating set. The dependence of the construction on the choice of generating set will be discussed in Section 3.

Many combinatorial constructions can be interpreted geometrically or topologically, and this often results in the most efficient means of proof. This is the main theme of this course. Nevertheless, the earlier combinatorial tools developed by Higman, Neumann etc, still remain a powerful resource, and a rich source of interesting examples.

2.1. Basic terminology and notation.

Let K be a graph. Formally this is thought of as a set, $V(K)$, of vertices together with a set $E(K)$ of edges. It will be convenient to allow multiple edges (edges connecting the same pair of vertices) and loops (edges starting and finishing at the same vertex). We recall some standard terminology from graph theory:

Definition : A (combinatorial) *path* consists of a sequence of edges with consecutive edges adjacent.

An *arc* is an embedded path.

A *cycle* is a closed path.

A *circuit* is an embedded cycle.

A graph is *connected* if every pair of vertices are connected by a path (and hence also by an arc).

The *valence* of a vertex is the number of incident edges (counting muliplicities of multiple edges, and counting each loop twice).

A graph is *locally finite* if each vertex has finite valence.

It is *n-regular* if every vertex has valence n.

We will often write a combinatorial path as a sequence of vertices rather than edges, though if there are loops or multiple edges one also needs to specify the edges connecting them.