1. Group presentations.

A group presentation gives a means of specifying a group up to isomorphism. It is the basis of the now "classical" combinatorial theory of groups. In Section 2, we will give a more geometrical intepretation of these constructions.

1.1. Notation.

Throughout the course, we will use the following fairly standard notation relating to groups.

$G \subseteq \Gamma$: G is a subset of Γ.

$G \leq \Gamma$: G is a subgroup of Γ.

$G \lhd \Gamma$: G is a normal subgroup of Γ.

$G \cong \Gamma$: G is isomorphic to Γ.

$1 \in \Gamma$ is the identity element of Γ.

$[\Gamma : G]$ is the index of G in Γ.

We write $|A|$ for the cardinality of a set A. In other words, $|A| = |B|$ means that there is a bijection between A and B. (This should not be confused with the fairly standard notation for "realisations" of complexes, used briefly in Section 2.)

We use **N**, **Z**, **Q**, **R**, **C** respectively for the natural numbers (including 0), the intergers, and the rational, real and complex numbers.

We shall generally view $\mathbf{Z}^n$ and $\mathbf{R}^n$ from different perspectives. We shall normally think of $\mathbf{Z}^n$ as group under addition, and $\mathbf{R}^n$ as a metric space with the euclidean norm.

1.2. Generating sets.

Let Γ be a group and $A \subseteq \Gamma$. Let $\langle A \rangle$ be the intersection of all subgroups of Γ containing the set A. Thus, $\langle A \rangle$ is the unique smallest subgroup of Γ containing the set A. In other words, it is characterised by the following three properties:

(G1) $A \subseteq \langle A \rangle$,

(G2) $\langle A \rangle \leq \Gamma$, and