4

Classifi cation of log del Pezzo surfaces of index ≤ 2 and applications

4.1. Classification of log del Pezzo surfaces of index ≤ 2

From the results of Chapters 1 — 3 we obtain

Theorem 4.1. *For any log del Pezzo surface Z of index ≤ 2 there exists a unique resolution of singularities $\sigma : Y \to Z$ (called* right*) such that Y is a right DPN surface of elliptic type and σ contracts exactly all exceptional curves of the Du Val and the logarithmic part of $\Gamma(Y)$. Vice versa, if Y is a right DPN surface of elliptic type, then there exists a unique morphism $\sigma : Y \to Z$ of contraction of all exceptional curves corresponding to the Du Val and the logarithmic part of $\Gamma(Y)$ which gives resolution of singularities of log del Pezzo surface Z of index ≤ 2 (it will be automatically the right resolution).*

Thus, classifications of log del Pezzo surfaces of index ≤ 2 and right DPN surfaces of elliptic type are equivalent, and they are given by Theorems 3.18, 3.19 and 3.20.

Proof. Let Z be a log del Pezzo surface of index ≤ 2. In Chapter 1, a "canonical" (i. e. uniquely defined) resolution of singularities $\sigma : Y \to Z$ had been suggested such that Y is a right DPN surface of elliptic type. First, a minimal resolution of singularities $\sigma_1 : Y' \to Z$ is taken, and second, the blow-up of all intersection points of components of curves in preimages of non Du Val singularities K_n of Z is taken. Let us show that σ contracts exactly exceptional curves of $\operatorname{Duv}\Gamma(Y)$ and $\operatorname{Log}\Gamma(Y)$.

Let E be an exceptional curve of Y corresponding to a vertex of the subgraph $\operatorname{Duv}\Gamma(Y)$ or $\operatorname{Log}\Gamma(Y)$. Let $\widetilde{C_g} \in |-2K_Z|$ be a non-singular curve of Z which does not contain singular points of Z (it does exist by Theorem 1.5), and $C_g = \sigma^{-1}(\widetilde{C_g})$. Then (see Sections 1.5 and 2) $C_g + E_1 + \cdots + E_k \in |-2K_Y|$ where E_i are all exceptional curves on Y with the square (-4) and C_g a non-singular irreducible curve of genus $g \geq 2$.