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General Theory of DPN surfaces and K3 surfaces with non-symplectic involution

2.1. General remarks

As it was shown in Chapter 1, a description of log del Pezzo surfaces of index ≤ 2 is reduced to a description of rational surfaces Y containing a nonsingular curve $C \in |-2K_Y|$ and a certain configuration of exceptional curves. Such surfaces Y and exceptional curves on them were studied in the papers [Nik79, Nik83, Nik84a, Nik87] of the second author. They are one of possible generalizations of del Pezzo surfaces.

Many other generalizations of del Pezzo surfaces were proposed, see e.g. [Dem80, Har85a, Har85b, Loo81], and most authors call their surfaces "generalized del Pezzo surfaces". Therefore, we decided following [Nik87] to call our generalization DPN surfaces. One can consider DPN surfaces to be some appropriate non-singular models of log del Pezzo surfaces of index ≤ 2 and some their natural generalizations.

Definition 2.1. A nonsingular projective algebraic surface Y is called a **DPN surface** if its irregularity $q(Y) = 0$, $K_Y \neq 0$ and there exists an effective divisor $C \in |-2K_Y|$ with only simple rational, i.e. A, D, E-singularities. Such a pair (Y, C) is called a **DPN pair.** A DPN surface Y is called **right** if there exists a nonsingular divisor $C \in |-2K_Y|$; in this case the pair (Y, C) is called **right DPN pair** or nonsingular DPN pair.

The classification of algebraic surfaces implies that if $C = \emptyset$ then a DPN surface Y is an Enriques surface ($\varkappa = p = q = 0$). If $C \neq \emptyset$ then Y is a rational surface ($\varkappa = -1, p = q = 0$), e.g. see [Shaf65].

Using the well-known properties of blowups, the following results are easy to prove. Let (Y, C) be a DPN pair, $E \subset Y$ be an exceptional curve of the 1st kind on Y and $\sigma : Y \to Y'$ be the contraction of E. Then $(Y', \sigma(C))$ is also a DPN pair. In this way, by contracting exceptional curves of the 1st kind, one can always arrive at a DPN pair (Y', C') where Y' is a relatively minimal (i.e. without exceptional curves of the 1st kind) rational surface. In