CHAPTER VII

Around log-terminal singularities

In this chapter, we discuss singularities arising from the consideration on the minimal model theory of higher-dimensional algebraic varieties. The notion of terminal singularities and that of canonical singularities are introduced by Reid in the study of singularities on minimal models ([**113**], [**114**]). In the minimal model program, we consider not only normal varieties themselves but also the pairs consisting of normal varieties and effective $\mathbb{Q}$-divisors. Notions of singularities can be defined similarly for such pairs. In the middle of 1980's, there appeared a summary [**61**] of minimal model program for higher dimensional varieties, where the notions of log-terminal, log-canonical, and weakly log-terminal are explained. The definition of log-terminal in [**61**] is different from the one used in the classification theory of open surfaces, in the sense that the latter allows a $\mathbb{Q}$-divisor with multiplicity one. Shokurov [**132**] introduced his original definition of log-terminal (it was written *log terminal*) in order to prove the log-flip conjectures, which coincides in dimension two with the one used in the classification theory of open surfaces. The notion of log-terminal in [**61**] is given a different name and called *Kawamata log terminal* or klt in [**132**] and [**74**]. However, Shokurov's notion of log terminal seems to have no good meaning for application. The notion of divisorial log terminal (dlt) in [**132**] and [**74**] is useful for the log minimal model program. In [**134**], the notion of dlt is shown to be equivalent to the notion of weakly log-terminal if we consider only simple normal crossing divisors in the definition given in [**61**]. Unfortunately, however, the notion of dlt is not a property well-defined for analytic germs. Fujita's definition of log terminal in [**27**] dealt with the analytic local situation. In the early 1990's, the author introduced another notion of log-terminal, named strongly log-canonical, which is closer to the notion of log-canonical. It is a property well-defined for analytic germs and has many useful properties for the minimal model program.

In this chapter, we introduce the notions of *admissible*, *quasi log-terminal*, and *strongly log-canonical*, for pairs (X, Δ) consisting of normal varieties and effective $\mathbb{R}$-divisors. These notions are analytically local in nature. These are defined and discussed in §**1**. In the definition of admissible pairs, the $\mathbb{R}$-divisor $K_X + \Delta$ need not to be $\mathbb{R}$-Cartier. A new proof of rationality of canonical singularities is also given in §**1**. The minimal model program for strongly log-canonical pairs is mentioned in §**2** and a relation between admissible singularities and ω-sheaves is explained in §**3**.