2 Definition

In this section, we derive the definition of viscosity solutions of (1.1) via the vanishing viscosity method.

We also give some basic properties of viscosity solutions and equivalent definitions using "semi-jets".

2.1 Vanishing viscosity method

When the notion of viscosity solutions was born, in order to explain the reason why we need it, many speakers started in their talks by giving the following typical example called the eikonal equation:

$$|Du|^2 = 1 \quad \text{in } \Omega. \tag{2.1}$$

We seek C^1 functions satisfying (2.1) under the Dirichlet condition:

$$u(x) = 0 \quad \text{for } x \in \partial\Omega. \tag{2.2}$$

However, since there is no classical solution of (2.1)-(2.2) (showing the non-existence of classical solutions is a good exercise), we intend to derive a reasonable definition of weak solutions of (2.1).

In fact, we expect that the following function (the distance from $\partial\Omega$) would be the unique solution of this problem (see Fig 2.1):

$$u(x) = \mathrm{dist}(x, \partial\Omega) := \inf_{y\in\partial\Omega} |x-y|.$$

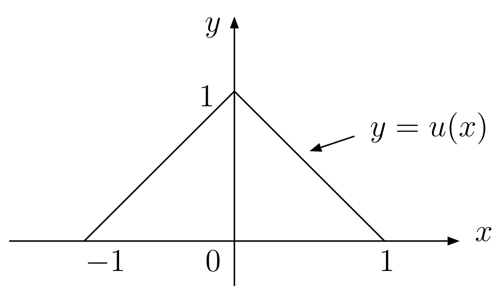

Fig 2.1