APPENDIX C

p-adic symmetric domains and Totaro's theorem

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This appendix is a short exposition of M. Rapoport and T. Zink's construction of p-adic symmetric domains [**RZ96**] and of B. Totaro's theorem [**Tot96**]. Let G be a connected reductive algebraic group over $\mathbb{Q}_p$. The set $\mathcal{F}$ of filtrations on an F-isocrystal with G-structure has a structure of a homogeneous space. Rapoport and Zink introduced a p-adic rigid analytic structure on the set $\mathcal{F}^{wa}$ of weakly admissible points in $\mathcal{F}$. They conjectured that the point in $\mathcal{F}^{wa}$ is characterized by the semistability in the sense of the geometric invariant theory [**MFK94**] and Totaro proved this conjecture.

1. Weakly admissible filtered isocrystals.

We recall J.-M. Fontaine's definition of weakly admissible filtered F-isocrystals [**Fon79**].

1.1. Let p be a prime number, k a perfect field of characteristic p, K_0 an absolutely unramified discrete valuation field of mixed characteristics $(0,p)$ with residue field k, $\overline{K}_0$ an algebraic closure of K_0, and σ the Frobenius automorphism on K_0.

Definition 1.2. (1) An F-isocrystal over k, (we simply say "isocrystal"), is a finite dimensional K_0-vector space V with a bijective σ-linear endomorphism $\Phi : V \to V$. We denote the category of isocrystals over k by $\mathrm{Isoc}(K_0)$.

(2) For a totally ramified finite extension K of K_0 in $\overline{K}_0$, a filtered isocrystal $(V, \Phi, F^\bullet)$ over K is an isocrystal (V, Φ) with a decreasing filtration $F^\bullet$ on the K-vector space $V \otimes_{K_0} K$ such that $F^r = V \otimes_{K_0} K$ for $r \ll 0$ and $F^s = 0$ for $s \gg 0$. We denote the category of filtered isocrystals over K by $MF(K)$.

Fontaine also introduced a filtered isocrystal with nilpotent operator N [**Fon94**]. In this appendix we restrict our attension to filtered isocrystals with $N = 0$.

The category $MF(K)$ is a $\mathbb{Q}_p$-linear additive category with $\otimes$ and internal Hom's, but not abelian. A subobject $(V', \Phi', F'^\bullet)$ of a filtered isocrystal