APPENDIX B

An overview of the theory of p-adic uniformization.

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In this appendix we always denote by K, R, and π a complete discrete valuation field, its valuation ring, and a prime element of R, respectively. We assume that the residue field $k = R/\pi R$ is finite and consists of q elements. The Mumford-Kurihara-Mustafin uniformization is a procedure to construct nice analytic (and in many cases algebraic) varieties by taking discrete Schottky-type quotients of a certain p-adic analogue of symmetric space, so-called *Drinfeld symmetric space* Ω, or its variants.

1. Bruhat-Tits building.

First we survey the construction of a certain simplicial complex, called Bruhat-Tits building (attached to $\mathrm{PGL}(n+1, K)$), which will be closely related with the Drinfeld symmetric space. Let n be a positive integer and V an $n+1$ dimensional vector space over K. A *lattice* in V is a finitely generated R-submodule of V which spans V over K. Every lattice is therefore a free R-module of rank $n+1$. Let $\widetilde{\Delta}_0$ be the set of all lattices in V. We say that two lattices M_1 and M_2 are *similar* if there exists $\lambda \in K^\times$ such that $M_1 = \lambda M_2$. The similarity is obviously an equivalence relation. We denote by Δ_0 the set of all similarity classes of lattices in V.

Definition 1.1. The *Bruhat-Tits building* (attached to $\mathrm{PGL}(n+1, K)$) is the finite dimensional simplicial complex Δ with the vertex set Δ_0 defined as follows: A finite subset $\{\Lambda_0, \ldots, \Lambda_l\}$ of Δ_0 forms an l-simplex if and only if, after permuting indices if necessary, one can choose $M_i \in \Lambda_i$ for $0 \leq i \leq n+1$ such that

$$M_0 \supsetneq M_1 \supsetneq \cdots \supsetneq M_l \supsetneq \pi M_0.$$

The role of M_0 is by no means important; for example, one can shift the indices like $M_1 \supsetneq \cdots \supsetneq M_l \supsetneq \pi M_0 \supsetneq \pi M_1$.

To understand the structure of Δ, let us fix one vertex $\Lambda = [M]$. Suppose $\{\Lambda_0, \ldots, \Lambda_l\}$ is an l-simplex having Λ as a vertex; we may assume $\Lambda = \Lambda_0$, and can take $M_i \in \Lambda_i$ for $1 \leq i \leq l$ such that $M \supsetneq M_1 \supsetneq \cdots \supsetneq M_l \supsetneq \pi M$. Set $\overline{M}_i := M_i/\pi M$. Then we get a flag

$$\overline{M} \supsetneq \overline{M}_1 \supsetneq \cdots \supsetneq \overline{M}_l \supsetneq 0$$