

The Littlewood-Richardson Rule, and Related Combinatorics

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Abstract

An introduction is given to the Littlewood-Richardson rule, and various combinatorial constructions related to it. We present a proof based on tableau switching, dual equivalence, and coplactic operations. We conclude with a section relating these fairly modern techniques to earlier work on the Littlewood-Richardson rule.

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