

Lectures on Noncommutative Symmetric Functions

Jean-Yves THIBON

Abstract

This is the text of lectures delivered at the RIMS (Kyoto University) in July 1998. It presents the basic structures of the theory of noncommutative symmetric functions, with emphasis on the parallel with the commutative theory and on the representation theoretical interpretations. Some examples involving descent algebras and characters of symmetric groups are discussed in detail.

Table of Contents

1	Introduction	40
2	Some highlights of the commutative theory	42
3	The Hopf algebra of noncommutative symmetric functions	47
3.1	Algebraic generators	47
3.2	Linear bases	49
3.3	Duality and quasi-symmetric functions	50
3.4	Descent algebras and internal product	53
4	Representation theoretical interpretations	59
4.1	The 0-Hecke algebra	59
4.2	The 0-quantum GL_n	67
4.3	The 0-quantized enveloping algebra $\mathcal{U}_0(gl_n)$	74
5	Selected applications	80
5.1	Lie idempotents and the Hausdorff series	80
5.2	Noncommutative cyclic characters of symmetric groups	84
5.3	Diagonalization of the left q -bracketing	86