CHAPTER 4

Definition of complements and elementary properties

4.1. Introduction

The following conjecture is called *Reid's general elephant conjecture*

CONJECTURE 4.1.1. *Let $f\colon X \to Z \ni o$ be a K_X-negative contraction from a threefold with only terminal singularities. Then near the fiber over o the linear system $-K_X$ contains a divisor having only Du Val singularities.*

At the moment it is known that this conjecture is true (only in analytic situation) in the following cases:

- $X = Z \ni o$ is an isolated singularity [**RY**], moreover, this is equivalent to the classification of three-dimensional terminal singularities;
- $f\colon X \to Z$ is an extremal flipping or divisorial small contraction [**Mo**], [**KoM**], this is a sufficient condition for the existence of flips [**K**].

Some particular results are known in the case when $f\colon X \to Z$ is an extremal contraction to a surface [**P**]. This case is interesting for applications to rationality problem of conic bundles.

However, at the moment it is not so clear how one can prove Reid's conjecture in the algebraic situation. Moreover, it fails for the case $Z = \mathrm{pt}$ (there are examples of $\mathbb{Q}$-Fano threefolds with empty $|-K_X|$). Shokurov proposed the notion of complements, which is weaker then "general elephant" but much more easier to work with.

DEFINITION 4.1.2. Let (X, D) be a log pair, where D is a subboundary. Then a *$\mathbb{Q}$-complement* of $K_X + D$ is a log divisor $K_X + D'$ such that $D' \geq D$, $K_X + D'$ is lc and $n(K_X + D') \sim 0$ for some $n \in \mathbb{N}$.

DEFINITION 4.1.3 ([**Sh2**]). Let X be a normal variety and $D = S + B$ a subboundary on X, such that B and S have no common components, S is an effective integral divisor and $\lfloor B \rfloor \leq 0$. Then we say that $K_X + D$ is *n-complementary*, if there is a $\mathbb{Q}$-divisor D^+ such that

(i) $n(K_X + D^+) \sim 0$ (in particular, nD^+ is integral divisor);
(ii) $K_X + D^+$ is lc;
(iii) $nD^+ \geq nS + \lfloor (n+1)B \rfloor$.