CHAPTER 1

Preliminary results

1.1. Singularities of pairs

List of notations.

$\equiv$	numerical equivalence
$\sim$	linear equivalence
$\sim_{\mathbb{Q}}$	$\mathbb{Q}$-linear equivalence
$\mathcal{K}(X)$	function field of X
$D \approx D'$	D and D' gives the same valuation of $\mathcal{K}(X)$
$\rho(X)$	Picard number of X, rank of the Néron-Severi group
$Z_1(X/Z)$	group of 1-cycles on X over Z (see [**KMM**])
$N_1(X/Z)$	quotient of $Z_1(X/Z)$ modulo numerical equivalence (cf.)
$\overline{NE}(X/Z)$	Mori cone (see [**KMM**])
$\mathrm{Weil}(X)$	group of Weil divisors, i.e., the free abelian group generated by prime divisors on X
$\mathrm{Weil}_{\mathrm{lin}}(X)$, $\mathrm{Weil}_{\mathrm{alg}}(X)$	quotients of $\mathrm{Weil}(X)$ modulo linear and algebraic equivalence respectively.

All varieties are assumed to be algebraic varieties defined over the field $\mathbb{C}$. By a *contraction* we mean a projective morphism $f\colon X \to Z$ of normal varieties such that $f_*\mathcal{O}_X = \mathcal{O}_Z$ (i.e., having connected fibers). We call a birational contraction a *blowdown* or *blowup*, depending on our choice of initial variety.

A *boundary* on a variety X is a $\mathbb{Q}$-Weil divisor $D = \sum d_i D_i$ with coefficients $0 \le d_i \le 1$. If we have only $d_i \le 1$, we say that D is a *subboundary.* All varieties are usually considered supplied with boundary (or subboundary) as an additional structure. If D is a boundary, then we say that (X, D) is a *log variety* or *log pair.* Moreover, if we have a contraction $f\colon X \to Z$, then we say that (X, D) is a *log variety* over Z and denote it simply by $(X/Z, D)$. If $\dim Z > 0$, we often consider Z as a germ near some point $o \in Z$. To specify this we denote the corresponding log variety by $(X/Z \ni o, D)$.

Given a birational morphism $f\colon X \to Y$, the boundary D_Y on Y is usually considered as the image of the boundary D_X on X: $D_Y = f_*D_X$. The integral part of a $\mathbb{Q}$-divisor $D = \sum d_i D_i$ is defined in the usual way: $\lfloor D \rfloor := \sum \lfloor d_i \rfloor D_i$, where $\lfloor d_i \rfloor$ is the greatest integer such that $\lfloor d_i \rfloor \le d_i$. The (round up) upper integral part $\lceil D \rceil$ and the fractional part $\{D\}$ are similarly defined.