3 Fourier transform and Sobolev spaces on flat space

3.1 Overview

In this Chapter we shall introduce two basic tools for the study of hyperbolic equations. Namely, we start with the Fourier transform and the main facts about Sobolev spaces in $\mathbf{R}^n$. We shall avoid a complete and detailed representation of the theory of Fourier transform and Sobolev spaces. Nevertheless, we shall underline only the points, which are important for a further generalization for the case of manifolds of curvature -1.

The reader can use [44], [21] , [43] [6] for more detailed information about the space of distributions, Fourier transform and the convolution.

3.2 Preliminary facts about holomorphic functions

Let $\mathbf{C}$ be the complex plane and let $U \subseteq \mathbf{C}$ be an open domain in this plane. Any point $z \in U$ can be represented as

$$z = x + iy,$$

where x, y are real numbers. A function

$$f : U \to \mathbf{C}$$

is $C^1(U)$ if the partial derivatives

$$\partial_x f(x+iy), \partial_y f(x+iy)$$

exist and are continuous functions. Of special interest are the vector fields

$$\partial_z = \frac{1}{2}(\partial_x - i\partial_y)$$

and

$$\partial_{\bar{z}} = \frac{1}{2}(\partial_x + i\partial_y).$$

If $f \in C^1(U)$, then f is called holomorphic in U, if satisfies the equation

$$\partial_{\bar{z}} f(z) = 0, \quad z \in U.$$

One can see that a function $f : U \to \mathbf{C}$ is holomorphic in U if and only if

$$\lim_{h \to 0} \frac{f(z+h) - f(z)}{h}$$

exists for any $z \in U$.