2 Preliminaries from functional analysis

2.1 Overview

In this chapter we shall make a review of some basic facts from functional analysis and we shall focus our attention to two main points.

On one hand, we shall give suitable sufficient conditions that assure that a symmetric strictly monotone operator in a Hilbert space is self-adjoint. More precisely, we consider Friedrich's extention of a symmetric strictly monotone operator. The criterion to assure that its closure is self-adjoint operator is of type: weak solution $\Rightarrow$ strong solution. We shall apply this criterion in the next chapters.

On the other hand, we represent some of the basic interpolation theorems for the Lebesgue spaces L^p.

To get a complete information on the subject one can use [42], [43] , [65].

2.2 Linear operators in Banach spaces

Given any couple A, B of Banach spaces we denote their corresponding norms by

$$\|a\|_A \quad , \quad \|b\|_B$$

for $a \in A, b \in B$. A linear operator

$$F : A \to B$$

is called bounded (or continuous) if there is a constant $C > 0$ such that

$$\|Fa\|_B \leq C\|a\|_A.$$

The space $L(A, B)$ is the set of bounded linear operators

$$F : A \to B$$

with norm

$$\|F\| = \sup_{\|a\|_A=1} \|Fa\|_B.$$

In case $A = B$ we shall denote by $L(A)$ the corresponding linear space of bounded linear operators from A in A. It is easy to see that $L(A, B)$ equipped with the above norm is a Banach space.

If B is the field $\mathbf{C}$ of complex numbers, then the elements in $L(A, \mathbf{C})$ are called functionals and $L(A, \mathbf{C})$ itself is called dual space of A and is denoted by A'.

For any $v' \in A'$ we denote by

$$< v', v >$$

the action of the linear functional v' on $v \in A$. There is a natural embedding

$$J : A \to A',$$