

Chapter 5

Homogeneous quasilinear hyperbolic systems

In Chapter 3 and Chapter 4 we systematically study the global existence and the blow-up phenomenon of C^1 solutions to Cauchy problem for general quasilinear hyperbolic system with small and decay initial data. However, the whole discussion is based on the existence of the normalized coordinates. Unfortunately, for the non-strictly hyperbolic case, in general we do not know if there exist the normalized coordinates, and even if the normalized coordinates exist, it is still very hard to check the hypotheses given in the normalized coordinates. Therefore, a consideration without the normalized coordinates is needed. Such a discussion for homogeneous quasilinear hyperbolic systems is carried out in this Chapter. Essentially restricting our system in such a way that each characteristic is either genuinely nonlinear or linearly degenerate in the sense of P.D.Lax, we only require the assumption that $\bar{\theta}$ (see (1.14)) is small instead of the hypothesis that θ (see (1.13)) is small, eliminate the use of the normalized coordinates and obtain more results including a sharp estimate on life span of C^1 solutions. In particular, we give a lower bound of life span of the C^1 solution to the Cauchy problem (1.1) and (1.7) for the small initial data with certain monotone properties. Moreover,