# Chapter 4

# Quasilinear non-strictly hyperbolic systems

In Chapter 3, we discussed the global existence and the blow-up phenomenon, particularly the life span and the breakdown behaviour of classical solutions to Cauchy problem for quasilinear strictly hyperbolic systems with small and decay initial data. This chapter aims to generalize the result presented in Chapter 3 to the case that system (1.1) might be non-strictly hyperbolic.

## §4.1. Generalized null condition

Consider quasilinear hyperbolic system (1.1), where we assume that the eigenvalues $\lambda_i(u)$ and left (resp. right) eigenvectors $l_i(u) = (l_{i1}(u), \cdots, l_{in}(u))$ (resp. $r_i(u) = (r_{i1}(u), \cdots, r_{in}(u))^T$ ) of $A(u)$ have the same regularity as $a_{ij}(u)$ $(i, j = 1, \cdots, n)$, and (1.4)-(1.6) holds. However, we do not require system (1.1) must be strictly hyperbolic.

Without loss of generality, we may suppose that

$$\lambda_0 \triangleq \lambda_1(0) = \cdots = \lambda_p(0) < \lambda_{p+1}(0) < \cdots < \lambda_n(0). \tag{4.1.1}$$